Effective interest rate

Use effective interest rate to calculate future or present value.

现金流支付频率为年，使用年有效利率

现金流支付频率为其他频率，使用相应的有效利率

n and r must be expressed in terms of the same time as the period between each cash flow.

n代表现金流笔数

Compounding

$$r_n = Annual\ Nominal\ Rate$$

$$r_p = Effective\ Periodic\ Rate$$

$$r_e = Effective\ Annual\ Rate$$

$$n = number\ of\ compounding\ periods\ per\ annum$$

$$r_p = \frac{r_n}{n}$$

$$r_e = \left(1 + r_p\right)^n - 1 = \left(1 + \frac{r_n}{n}\right)^n - 1$$

如果 compounded annually，不需要考虑复利过程，$r_e = r_n$

r_p针对每一个复利期是有效的

r_e针对每一年是有效的

其他有效利率可以使用r_e或r_p进行计算

现金流支付频率为年

$$r = r_e$$

现金流支付频率为复利周期

$$r = r_p$$

现金流支付频率为其他频率，使用相应的有效利率（通用方案）

$$r = (1 + r_e)^{\frac{1}{m}} - 1 = \left(1 + \frac{r_n}{n}\right)^{\frac{n}{m}} - 1$$

$$m = \text{cash flow payment frequency in one year}$$

F = 100 per year, n = 5 years, r = 10% p.a. compounded annually.

现金流支付频率为年

$$r = r_e = 0.1$$

F = 100 per month, n = 5 years, r = 12% p.a. compounded monthly.

现金流支付频率为复利周期

$$r = r_p = \frac{r_n}{n} = \frac{0.12}{12} = 0.01$$

或现金流支付频率为其他频率（月）

$$r = \left(1 + \frac{r_n}{n}\right)^{\frac{n}{m}} - 1 = \left(1 + \frac{0.12}{12}\right)^{\frac{12}{12}} - 1 = 0.01$$

F = 100 per year, n = 5 years, r = 12% p.a. compounded monthly.

现金流支付频率为年

$$r = r_e = \left(1 + \frac{r_n}{n}\right)^{n} - 1 = \left(1 + \frac{0.12}{12}\right)^{12} - 1 = 0.12682503$$

F = 100 per quarter, n = 5 years, r = 10% p.a. compounded annually.

现金流支付频率为其他频率（季度）

$$r = (1 + r_e)^{\frac{1}{m}} - 1 = (1 + 0.1)^{\frac{1}{4}} - 1 = 0.02411369$$

F = 100 per quarter, n = 5 years, r = 12% p.a. compounded monthly.

现金流支付频率为其他频率（季度）

$$r = \left(1 + \frac{r_n}{n}\right)^{\frac{n}{m}} - 1 = \left(1 + \frac{0.12}{12}\right)^{\frac{12}{4}} - 1 = 0.030301$$

A woman invests $1,000 at 10% p.a. compounded annually and plans to hold this investment for five years. How much will she have at the end of her holding period?

单笔现金流，现金流支付频率任意，可以使用年

$$r = r_e = 0.1$$

$$FV = 1,000 \times (1 + 0.1)^5 = 1,610.51$$

If you wish to provide $20,000 for your newborn's University education, how much should you invest now, given the interest rate that will accrue on the investment is 10% p.a. compounded monthly?

单笔现金流，现金流支付频率任意，可以使用年

$$r = r_e = \left(1 + \frac{r_n}{n}\right)^n - 1 = \left(1 + \frac{0.1}{12}\right)^{12} - 1 = 0.10471307$$

$$PV = 20,000 \times (1 + 0.10471307)^{-18} = 3,330.73$$

A company needs $10,000 in 5 years to replace a piece of equipment. How much must be invested now at an interest rate of 8% p.a. compounded daily in order to provide for this replacement?

单笔现金流，现金流支付频率任意，可以使用年

$$r = r_e = \left(1 + \frac{r_n}{n}\right)^n - 1 = \left(1 + \frac{0.08}{365}\right)^{365} - 1 = 0.08327757$$

$$PV = 10,000 \times (1 + 0.08327757)^{-5} = 6,703.49$$

A company needs $10,000 in 5 years to replace a piece of equipment. How much must be invested each year at 8% p.a. compounded semi-annually in order to provide for this replacement?

年金现金流，现金流支付频率为年

$$r = r_e = \left(1 + \frac{r_n}{n}\right)^n - 1 = \left(1 + \frac{0.08}{2}\right)^2 - 1 = 0.0816$$

$$FV = F \times \frac{(1 + 0.0816)^5 - 1}{0.0816} = 10,000$$

$$F = 1,699.14$$

A woman wants to provide a $3,000 university scholarship every year for ten years. The first scholarship is to be awarded one year from now. If the university can earn a 4% p.a. compounded monthly as a return on their investments, how much should the woman give now?

年金现金流，现金流支付频率为年

$$r = r_e = \left(1 + \frac{r_n}{n}\right)^n - 1 = \left(1 + \frac{0.04}{12}\right)^{12} - 1 = 0.04074154$$

$$PV = 3,000 \times \frac{1 - (1 + 0.04074154)^{-10}}{0.04074154} = 24,243.11$$

Annual sales revenue for your division was $2 million last year. Further, you expect that these revenues will grow indefinitely at a rate of 10% p.a. What is the present value of sales if the appropriate interest rate is 12% p.a. compounded annually?

年金现金流，现金流支付频率为年

$$r = r_e = 0.12$$

$$PV = \frac{2,000,000 \times (1 + 0.1)}{0.12 - 0.1} = 110,000,000$$